Algebraic solitary-wave polaritons in far-infrared transients

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We show that a single far-infrared pulse interacting with transverse-optic phonons in a dispersive dielectric offers a unique type of algebraic (rational) solitary wave in the presence of second- and third-order nonlinearities.

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It is now well known that many phenomena in contemporary physics can be explained in terms of solitons [1]. In optics, of a variety of model equations that predict the existence of solitonic solutions, along with the Korteweg–de Vries (KdV) and the sine-Gordon equations, the type that can be described by a family of nonlinear Schrödinger equations will be representative. With respect to the topological structure, the transverse configuration, and the asymptotic behavior in the tail, several classification methods of solitonlike fields are possible. For instance, the second classification consists of three categories: a bright, dark, and kink (shock-wave) type. The dark type is classifiable further into a gray, black, and darker-than-black solution [2]. A classification is also possible in terms of whether or not the field profile is describable with a combination of exponential functions, such as a hyperbolic function. One may find that the majority of solitary waves that have been explored so far are attributable to this family. An exception will be seen in what we call algebraic (or rational) solitons. As the term indicates, the field distribution of the algebraic solitons is expressed by a rational function such as a Lorentzian, and thus they are localized more weakly than the familiar hyperbolic-type solitons. To date the Lorentzian-type solitons (quasisolitons) have been found to exist in various physical contexts [3–12]. In the context of nonlinear optics, Mills presented a Lorentzian-shaped solitonlike solution, termed a type-II gap soliton, which could be sustained in nonlinear periodic structures within a frequency range near the lower bound of the gap [7]. In an analogous context, Grimshaw and Malomed recently showed that at a certain value of the wave velocity, gap solitons in a coupled KdV wave system degenerate into algebraic solitons with a Lorentzian intensity profile [8]. In the study of self-induced transparency of intensified radiation in a three-level medium, it was pointed out by Belenov and Poluetztov that a radiation pulse with the Lorentzian-shape intensity could propagate undistorted in the presence of two-photon resonances [9]. Hanamura attempted to modify their theory and arrived at the conclusion that, in the same situation, only the semistable Lorentzian pulse is possible [10]. Only recently have Kaplan, Shkolnikov, and Akanaev suggested conditions for experimental excitation of Lorentzian envelope solitons [11] that arise in stimulated Raman scattering [12]. In this paper we show analytically that in the presence of the second- and the third-order nonlinearities a single far-infrared pulse weakly interacting with transverse-optic (TO) phonons in a crystal medium offers an algebraic solitary wave. Both bright and dark polaritons are presented. In the latter we find that there exist two radically different types: one is nontopological (without phase shift along a transverse axis), the other is topological. The total phase shift of the topological solitary wave versus the confinement axis exceeds π. Analytical expressions for the cross-sectional shape of a variety of solitary-wave polariton pulses are presented.

We consider a situation where an intense electromagnetic field is coupled to TO phonons of a crystal medium [13]. The two lowest-order (i.e., $\chi^{(2)}$ and $\chi^{(3)}$) nonlinearities are assumed to give the nontrivial contribution to perturbing the electric displacement. When the center frequency ω is much less than the resonant frequency of the TO phonon (ω_{TO}) and the damping rate of the ac dielectric constant is negligible ($\Gamma \ll \omega \ll \omega_{\mathrm{TO}}$), the time-domain expression of the total electric displacement can be written in the form

$$D(t)=\varepsilon(0)E-(\Delta/\omega_{\mathrm{TO}}^2)E_{tt}+\chi^{(2)}E^2+\chi^{(3)}E^3 , \qquad (1)$$

where $\varepsilon(0)$ is the dc dielectric constant (the argument of ε indicates the frequency ω of radiation), $\Delta=\varepsilon(0)-\varepsilon(\infty)$, $\varepsilon(\infty)$ is the optical dielectric constant, and $\chi^{(2)}$ and $\chi^{(3)}$ are the coefficients of the quadratic and the cubic nonlinearities, respectively. (We assume in this paper that the magnitude of the coefficients is independent of ω.) Here we would like to stress that the addition of the cubic nonlinearity in our theoretical model will be reasonable because in many real materials, close correlation was predicted between $|\chi^{(2)}|$ and $|\chi^{(3)}|$ [14]. In other words, a crystal medium that exhibits an enhanced second-order nonlinearity tends to accompany a relatively large third-order nonlinearity. On substitution of Eq. (1) into the driving term in the (1+1)-dimensional scalar wave equation that can be derived from Maxwell equations, the nonlinear wave equation is derivable:

$$E_{zz}=c_0^{-2}[\varepsilon(0)E-(\Delta/\omega_{\mathrm{TO}}^2)E_{tt}+\chi^{(2)}E^2+\chi^{(3)}E^3]_{tt} , \qquad (2)$$

where c_0 is the speed of light in vacuum, and any transverse effects that will be responsible for additional diffraction terms E_{xx} and E_{yy} on the left-hand side of Eq. (2) have been ignored. With the following replacement (scaling)

$$E \to 6\varepsilon(0)(\chi^{(2)})^{-1}E \ , \tag{3a}$$

$$t \to [\Delta/\varepsilon(0)]^{1/2}\omega_{\mathrm{TO}}^{-1}t \ , \tag{3b}$$

$$z \to c_0\Delta^{1/2}[\varepsilon(0)\omega_{\mathrm{TO}}]^{-1}z \ , \tag{3c}$$

Eq. (2) can be reduced to

$$E_{zz} = E_{tt} - E_{tttt} + 6(E^2)_{tt} - \theta(E^3)_{tt} \ , \tag{4}$$

with

$$\theta = -36\chi^{(3)}\varepsilon(0)/(\chi^{(2)})^2 \ . \tag{5}$$

In an effort to obtain a traveling-wave solution in a frame of reference that moves with the traveling velocity of the electromagnetic pulse, we perform the coordinate transformation as

$$t \to t - \gamma z, \quad z \to z \ , \tag{6}$$

where γ is a real parameter that represents the reciprocal signal velocity. With this transformation from the global to the local coordinate, Eq. (4) is modified in the form

$$E_{zz} - 2\gamma E_{tz} + E_{tttt} + (\gamma^2 - 1)E_{tt} - 6(E^2)_{tt} + \theta(E^3)_{tt} = 0 \ . \tag{7}$$

This equation is a family of the Boussinesq equations [15], for which nonalgebraic solitary-wave solutions that include kink solitons were presented and discussed in various contexts [3,13,15–17]. In what follows we concentrate on eigensolutions along the propagation (the z) axis. Imposing the stationary requirement $\partial/\partial z \equiv 0$ on Eq. (7) leads to

$$E_{tttt} - \Lambda E_{tt} - 6(E^2)_{tt} + \theta(E^3)_{tt} = 0 \ , \tag{8}$$

where $\Lambda \equiv 1 - \gamma^2$.

As a nonsingular algebraic solitary-wave ansatz of Eq. (8) we shall set

$$E(t) = E_h(\alpha t^p + 1)^{-q} + E_b \ . \tag{9}$$

Here because E is not an envelope field but is the real electric field, E_h, E_b, α, p, and q must be real, which will be related to the constants in Eq. (8); to avoid singularities we assume that α, p, and q are positive. Note that $E(0) \equiv E_0 = E_h + E_b$ and $E(\pm\infty) = E_b$. On substitution of Eq. (9) into Eq. (8) we have obtained *solely for $p=2$ and $q=1$* a set of consistent relations

$$12\alpha E_h^{-1}E_b(1 + 2E_h^{-1}E_b) = -\Lambda \ , \tag{10a}$$

$$-6\alpha E_h^{-1}(1 + 4E_h^{-1}E_b) = -6 \ , \tag{10b}$$

$$8\alpha E_h^{-2} = \theta \ . \tag{10c}$$

Through direct substitution of Eq. (9) into Eq. (8), we have ensured that for any other combinations of p and q, Eq. (9) cannot be a solution of Eq. (8). With Eqs. (10) for $(p,q) = (2,1)$ one can readily solve for the three unknowns:

$$\alpha = \theta E_h^2/8 \ , \tag{11a}$$

$$E_h = 8/\theta - 4E_b \ , \tag{11b}$$

$$E_b = (3\theta)^{-1}[6 \pm (36 + 3\theta\Lambda)^{1/2}] \ . \tag{11c}$$

It can be concluded from Eq. (11a) that θ must be positive, which results, from Eq. (5), in $\chi^{(3)} < 0$. Since E_b must be real, it is necessary from Eq. (11c) that

$$|\gamma| \le (1 + 12/\theta)^{1/2} \ . \tag{12}$$

From Eqs. (11b) and (11c) the center amplitude becomes

$$E_0 = -\theta^{-1}[-2 \pm (36 + 3\theta\Lambda)^{1/2}] \ . \tag{13}$$

It should be noted from Eqs. (11) and (13) that there are two combinations for the shape parameters. Below we discuss each case separately.

Case I. Choose the upper sign of Eqs. (11c) and (13). From these equations we find immediately that $E_b > 0$ and $E_0 < E_b$. As is found from Eq. (13) the sign of E_0 depends on the velocity as

$$E_0 \ge 0 \quad \text{for } [1 + 32/(3\theta)]^{1/2} \le |\gamma| \le (1 + 12/\theta)^{1/2} \ , \tag{14a}$$

$$E_0 < 0 \quad \text{for } |\gamma| < [1 + 32/(3\theta)]^{1/2} \ . \tag{14b}$$

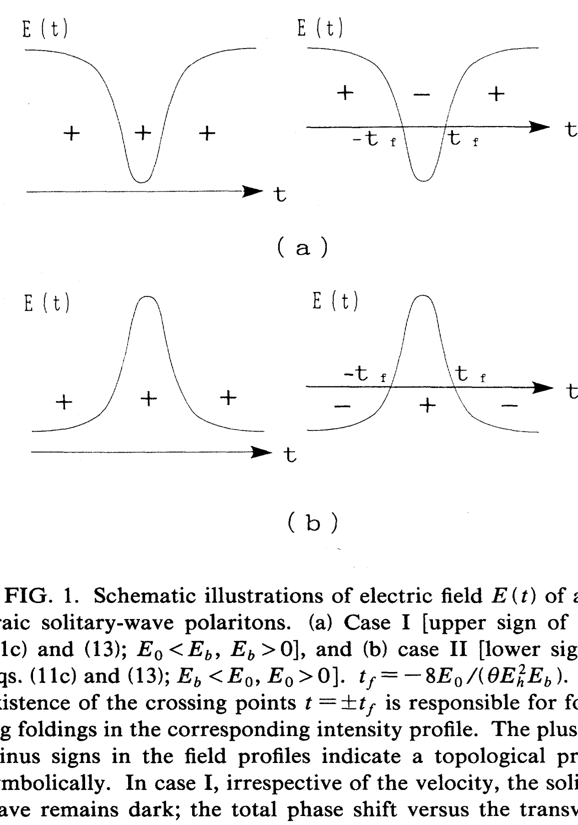

FIG. 1. Schematic illustrations of electric field $E(t)$ of algebraic solitary-wave polaritons. (a) Case I [upper sign of Eqs. (11c) and (13); $E_0 < E_b$, $E_b > 0$], and (b) case II [lower sign of Eqs. (11c) and (13); $E_b < E_0$, $E_0 > 0$]. $t_f = -8E_0/(\theta E_h^2 E_b)$. The existence of the crossing points $t = \pm t_f$ is responsible for forming foldings in the corresponding intensity profile. The plus and minus signs in the field profiles indicate a topological profile symbolically. In case I, irrespective of the velocity, the solitary wave remains dark; the total phase shift versus the transverse axis is dependent on the velocity. In contrast to this, in case II the wave is dark for the fast pulse ($|\gamma| < 1$), whereas it becomes bright for the slow pulse ($|\gamma| \ge 1$).

Case II. Choose the lower sign of Eqs. (11c) and (13). From these equations we find that $E_0 > 0$ and $E_b < E_0$. From Eq. (11c) the sign of E_b depends on the velocity as

$$E_b \geq 0 \quad \text{for } 1 \leq |\gamma| \leq (1+12/\theta)^{1/2} , \tag{15a}$$

$$E_b < 0 \quad \text{for } |\gamma| < 1 . \tag{15b}$$

Note here that the condition of Eq. (15b) is superluminous.

For both cases, schematic illustrations of the electric-field profile $E(t)$ are shown in Fig. 1, wherein one can see some unique features of the present solitary-wave pulse. First it is quite interesting to note from Fig. 1(b) that in case II there exist both bright- and dark-type solitary-wave polaritons. We notice in the topological solutions of Figs. 1(a) and 1(b) that there are twin intensity holes (a black hole doublet) at the two folding points, $|t_f| = -8E_0/(\theta E_h^2 E_b)$, and the topology changes abruptly from a minus $(-)$ to a plus $(+)$ and vice versa [18]. This behavior is of great interest because the total phase shift versus the transverse (the t) axis exceeds π. According to the terminology of Królikowski, Akhmediev, and Luther-Davies [2], such a topological solitary wave with an excessive phase shift can be classified into darker-than-black solitary waves.

Finally we shall discuss the stability of the polaritons. To examine a criterion for stability we define a mass integral

$$N \equiv \int_{-\infty}^{\infty} [E(t) - E_b]^2 dt = \pi (2/\theta)^{1/2} |E_h| , \tag{16}$$

where Eqs. (9) and (11a) have been used. From Eqs. (11b), (11c), and the relation $E_h = E_0 - E_b$, we express Λ in Eq. (8) with N:

$$\Lambda = (3\theta)^{-1} \{ [(3/\pi)(\theta/2)^{3/2} N]^2 - 36 \} . \tag{17}$$

According to the Vakhitov-Kolokolov criterion (N theorem) [19] the necessary condition for stability of a solitary-wave solution is given by $dN/d\Lambda > 0$. We find from Eq. (17) that $dN/d\Lambda > 0$ because $\theta > 0$. This result suggests that the algebraic solitary-wave polaritons are stable against propagation.

In conclusion, we have predicted that in the simultaneous presence of the second- and the third-order nonlinearities a far-infrared pulse interacting weakly with TO phonons in a dispersive dielectric could propagate as an algebraic solitary wave. The solitary wave has been found to possess unique properties, such as formation of noninteracting twin holes in the transverse intensity profile and realization of an excessive phase shift.

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